

## Two Comparative Case Studies of $b$ -quark and $c$ -quark Physics:

- Particle-antiparticle Mixing
- Radiative Weak Decays <sup>1</sup>

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### Abstract

We discuss prospects for detecting the two charm-related phenomena of  $D^0$ - $\bar{D}^0$  mixing and weak radiative decays of  $D$  mesons (*e.g.*  $D \rightarrow K^* + \gamma$ ). A general update of particle-antiparticle mixing for the pseudoscalar mesons is presented and the dynamics of mixing is reviewed, with application especially to the  $D^0$ - $\bar{D}^0$  system. The radiative weak decays of  $B$  mesons is then considered, and the problem of hadronic uncertainties is reviewed. Finally, the technique of calculating radiative weak decays for charm mesons is explained.

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Even with the recent discovery of the  $t$ -quark, we are clearly in the midst of the ‘ $b$ -quark era’. Two  $B$ -factories are under construction and extensive data samples have been produced at fixed-target accelerators and  $e^+e^-$  colliders. Although we all await the day when studies of CP-violation can be carried out, two discoveries already of special prominence are  $B_d$ - $\bar{B}_d$  mixing and the  $B \rightarrow K^*\gamma$  rare decay. At the same time, however, there has been an impressive advance in  $c$ -quark studies. Yet neither mixing nor weak radiative decays have been found for  $D$  mesons. We shall discuss the current status of both phenomena, emphasizing the differences between  $B$ -meson and  $D$ -meson systems.

### Particle-antiparticle Mixing

For simplicity, let us ignore the complication of CP-violation. Then mixing occurs if some interaction can convert a neutral pseudoscalar meson  $P^0$  to its antiparticle  $\bar{P}^0$ . If so, the two eigenstates  $P^{(1)}$  (CP-even) and  $P^{(2)}$  (CP-odd) experience a mass difference  $\Delta m$ , and oscillations between particle and antiparticle ensue (with  $\omega = \Delta m/2$ ) from an initial state of  $P^0$  (or  $\bar{P}^0$ ). Since mesons are unstable, decay also takes place. Let us denote the larger decay width of the two CP eigenstates by  $\Gamma$ . The competition between mixing and decay is characterized by parameter  $x \equiv \Delta m/\Gamma$ :

Table 1 Current Status of Mixing<sup>[1]</sup>

Meson	$K^0$	$B_d$	$D^0$	$B_s$
$x$	0.476 (1%)	0.71 (14%)	$< 0.083$	$> 2.0$

We see that two mixing parameters are accurately known, but the other two are only bounded. To grasp the physical meaning of these numbers, I recommend plotting  $\mathcal{I}m [(\Gamma/2)/(m - E - i\Gamma/2)]$  for each CP eigenstate. This is simply the line shape as given by the imaginary part of a Breit-Wigner function. The graphs accompanying this paper display (for all the meson systems) the profile of each CP-eigenstate as a function of energy, with unit of energy chosen to yield a Lorentzian shape of reasonable width. Let us comment briefly on each graph:

1.  $K^{(1)}$ - $K^{(2)}$ : Note the striking difference in widths. The CP-odd  $K^{(2)}$  is narrow due to suppressed phase space. The spacing between the

curves,  $\Delta m_K$ , is relatively well understood in terms of the quark box diagram, with some uncertainty associated with the precise value of the  $B_K$  parameter and the role of long-range effects. The feature least understood is the width of the broad CP-even  $K^{(1)}$ , which is the problem of the  $\Delta I = 1/2$  rule.

2.  $B_d^{(1)}$ - $B_d^{(2)}$ : The two curves have almost the same width since phase space is no longer an issue and there is no dynamical mechanism in the Standard Model to produce a large  $\Delta\Gamma$ . Compared to kaon mixing, all aspects of this diagram are well understood.
3.  $D^{(1)}$ - $D^{(2)}$ : Although one of the profiles appears to have been omitted, this is not the case. In the Standard Model,  $D$ -meson mixing is quite feeble and on a plot which displays the decay width in a natural manner, it is not possible to separate the  $D^{(1)}$  and  $D^{(2)}$  peaks!
4.  $B_s^{(1)}$ - $B_s^{(2)}$ : This case is almost opposite to the above since the mixing oscillation is expected to dominate decay. We use equal widths here, although  $\Delta\Gamma/\Gamma$  might be in the  $0.1 \rightarrow 0.2$  range.<sup>[2]</sup>

Of course, plotting the particle and antiparticle time dependence as oscillation/decay occurs is also instructive but limitation of space prevents us from doing so here.

What is the dynamics of mixing? Two categories of effects occur, short-range (quark box-diagrams) and long-range. Despite uncertainties in estimating meson decay constants and  $B$ -parameters, it is believed that short-range contributions are the most important component of all mixing amplitudes except perhaps for charm. As regards  $\Delta m_D$ , the current experimental bound and the value of the short-distance component are respectively

$$|\Delta m_D|^{(\text{expt})} < 1.3 \times 10^{-10} \text{ MeV} \quad \text{and} \quad |\Delta m_D|^{(\text{s.d.})} \simeq 0.8 \times 10^{-14} \text{ MeV} , \quad (1)$$

implying a gap of about four orders of magnitude! But there are also Standard Model long-range effects. Studies of the one-particle (*pole*) and some two-particle (*dispersive*) intermediate states imply long-distance values in the range  $|\Delta m_D|^{(\text{l.d.})} \sim 10^{-13} \text{ MeV}$ .<sup>[3]</sup>

Yet another possible theoretical approach to  $D^0$ - $\bar{D}^0$  mixing is application of heavy-quark effective theory. One expands in inverse powers of the  $c$ -quark

mass, and generates contributions  $|\Delta m_D|_{\text{HQET}}^{(n)}$  for the lowest orders  $n = 4, 6, 8$ . Although there is concern about using HQET at such low energies, we cite the results (in units of  $10^{-14}$  MeV)  $|\Delta m_D|_{\text{HQET}}^{(n)} \simeq (0.5 \rightarrow 0.9)$ ,  $(0.7 \rightarrow 2.0)$ ,  $(0.1 \rightarrow 0.6)$  respectively for  $n = 4, 6, 8$ .<sup>[4]</sup> These are smaller than the long-range estimates just given, and suggest possible cancellations between the various  $n$ -particle intermediate states.

## Radiative Weak Decays

Within errors, the recent observations of the exclusive decay  $B \rightarrow K^* \gamma$  and the associated inclusive  $b \rightarrow s \gamma$  transition<sup>[5]</sup> are both in accord with theoretical expectations (also having error bars!) of the Standard Model. These findings are important because of the dominance (due to the large  $t$ -quark mass) of the loop (*penguin*) amplitude. But there is important work yet to do. On the experimental side, the respective uncertainties of 39% and 29% must be decreased, while theoretically, advances must occur in computing QCD corrections and in taking the hadronic matrix element of the QCD-improved penguin operator.

When meaningful comparison between experiment and theory becomes a reality, the two will either agree or disagree. Then, either one cites success of the Standard Model and places limits on models of new physics<sup>[6]</sup> or one claims observation of new physics. I wish to caution against premature acceptance of the latter because the penguin is not the *only* contribution coming from the Standard Model — there can be nonspectator contributions as well as those from long range (nonpenguin) effects. In Ref. [7], a careful treatment is given for constructing a gauge-invariant vector-dominance (VMD) contribution to  $B \rightarrow K^* \gamma$ . Of course, such long-range effects are difficult to quantitatively estimate and one will see a number of increasingly sophisticated calculations appearing over time.

The theoretical picture of weak radiative decays in charm is quite different. The penguin amplitude is tiny, as shown in Table 2, which lists the intermediate-quark mass dependence of the Inami-Lim<sup>[8]</sup> function  $F_2$ , first without and then with the CKM dependence.

Table 2 Loops in $b \rightarrow s \gamma$ and $c \rightarrow u \gamma$		
$b \rightarrow s \gamma$	$F_2$	$V_{ib} V_{is}^* F_2$
$u$	$2.27 \times 10^{-9}$	$1.29 \times 10^{-12}$
$c$	$2.03 \times 10^{-4}$	$7.34 \times 10^{-6}$
$t$	0.39	$1.56 \times 10^{-2}$
$c \rightarrow u \gamma$	$F_2$	$V_{ci} V_{ui}^* F_2$
$d$	$1.57 \times 10^{-9}$	$3.36 \times 10^{-10}$
$s$	$2.92 \times 10^{-7}$	$6.26 \times 10^{-8}$
$b$	$3.31 \times 10^{-4}$	$3.17 \times 10^{-8}$

The remarkable influence of the  $t$ -quark mass in  $b \rightarrow s\gamma$  is evident, as is also the tiny magnitude of the Inami-Lin function for  $c \rightarrow u\gamma$ . A laborious calculation of the QCD-corrected hamiltonian for  $c \rightarrow u\gamma$  has been performed in Ref. [9] and even with a large QCD enhancement, the penguin component to  $c \rightarrow u\gamma$  is found to be negligible. Thus, long-range effects will surely dominate weak radiative decays of charm mesons and the pattern of observed decays can be expected to provide a stern test of our calculational abilities. At present, one has only limits such as  $B_{D^0 \rightarrow \rho^0 \gamma} < 1.4 \times 10^{-4}$  and  $B_{D^0 \rightarrow \phi^0 \gamma} < 2.0 \times 10^{-4}$ .<sup>[10]</sup> A large number of radiative weak decays of charm mesons is addressed in Ref. [9] and the modes  $D_s^+ \rightarrow \rho^0 \gamma$ ,  $D^0 \rightarrow K^{*0} \gamma$  are especially favored, with branching ratios in the  $10^{-5} \rightarrow 10^{-4}$  range. Unfortunately, these calculations are even more difficult than for  $B$ -mesons due to the presence of final-state interactions in the  $c$ -quark mass region.

## Conclusions

The very fact that neither mixing nor radiative weak decay have yet been observed in  $c$ -quark systems make these phenomena inviting targets for experimental study. In this talk, we have attempted to provide an update for each topic.

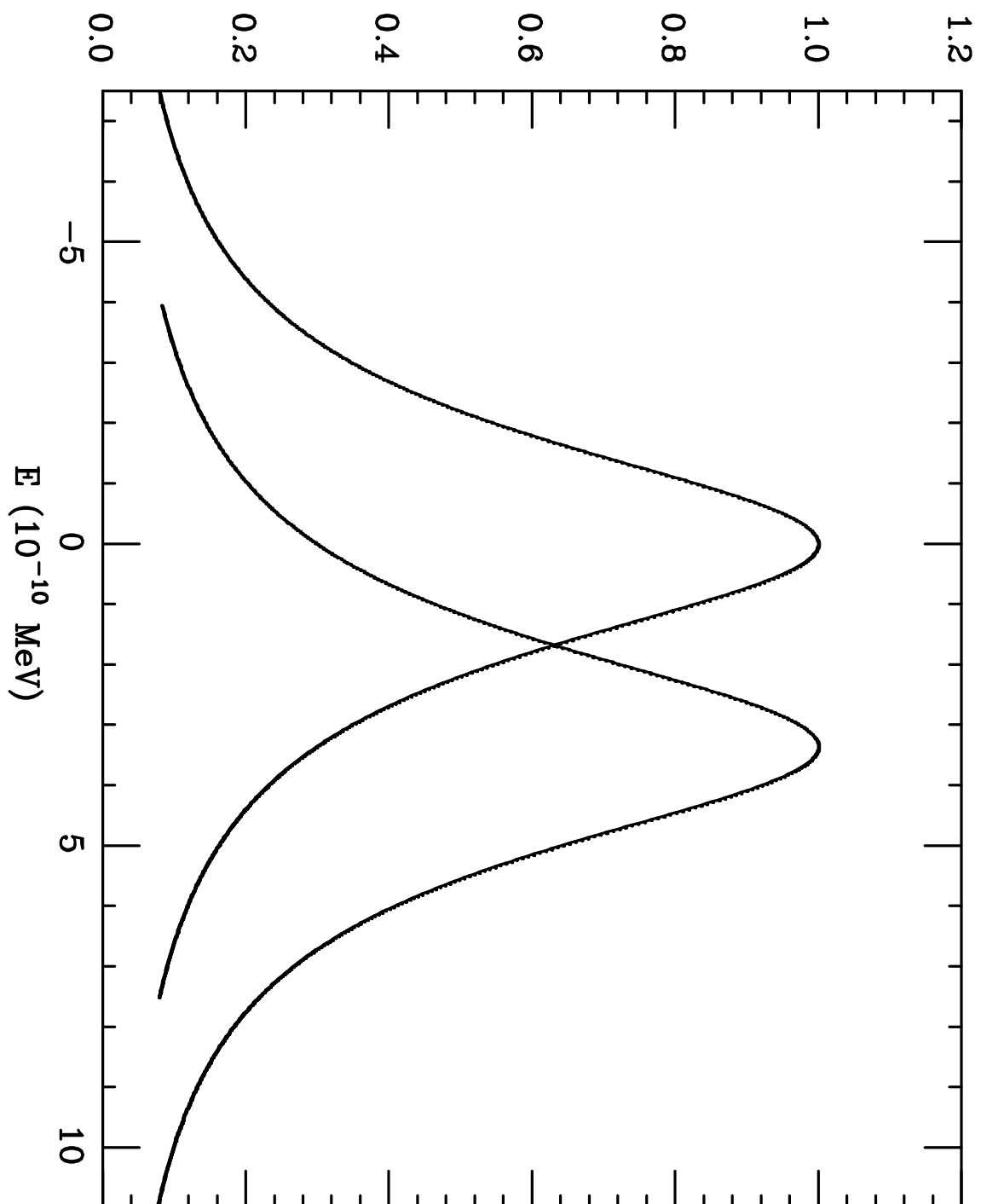
As regards  $D^0$ - $\bar{D}^0$  mixing, we feel that any experimental determination of  $\Delta m_D$  much larger than  $10^{-13}$  MeV would be grounds for excitement since some kind of new physics would presumably be responsible. It is the very suppression in the Standard Model of weak-mixing for charm that makes this a potentially rewarding area for study. However, the level of experimental sensitivity required will be exceedingly severe!

The situation is brighter for the near-term observation of radiative weak decays for  $D$  mesons, particularly at CLEO with its capability for detection of photons and neutral mesons. Rather than placing bounds on new physics, such findings will undoubtedly be of most interest in revealing the interplay of weak and QCD effects in the dynamically interesting charm quark region. In particular, they could yield significant insights as to the influence of final-state interactions.<sup>[9]</sup>

## References

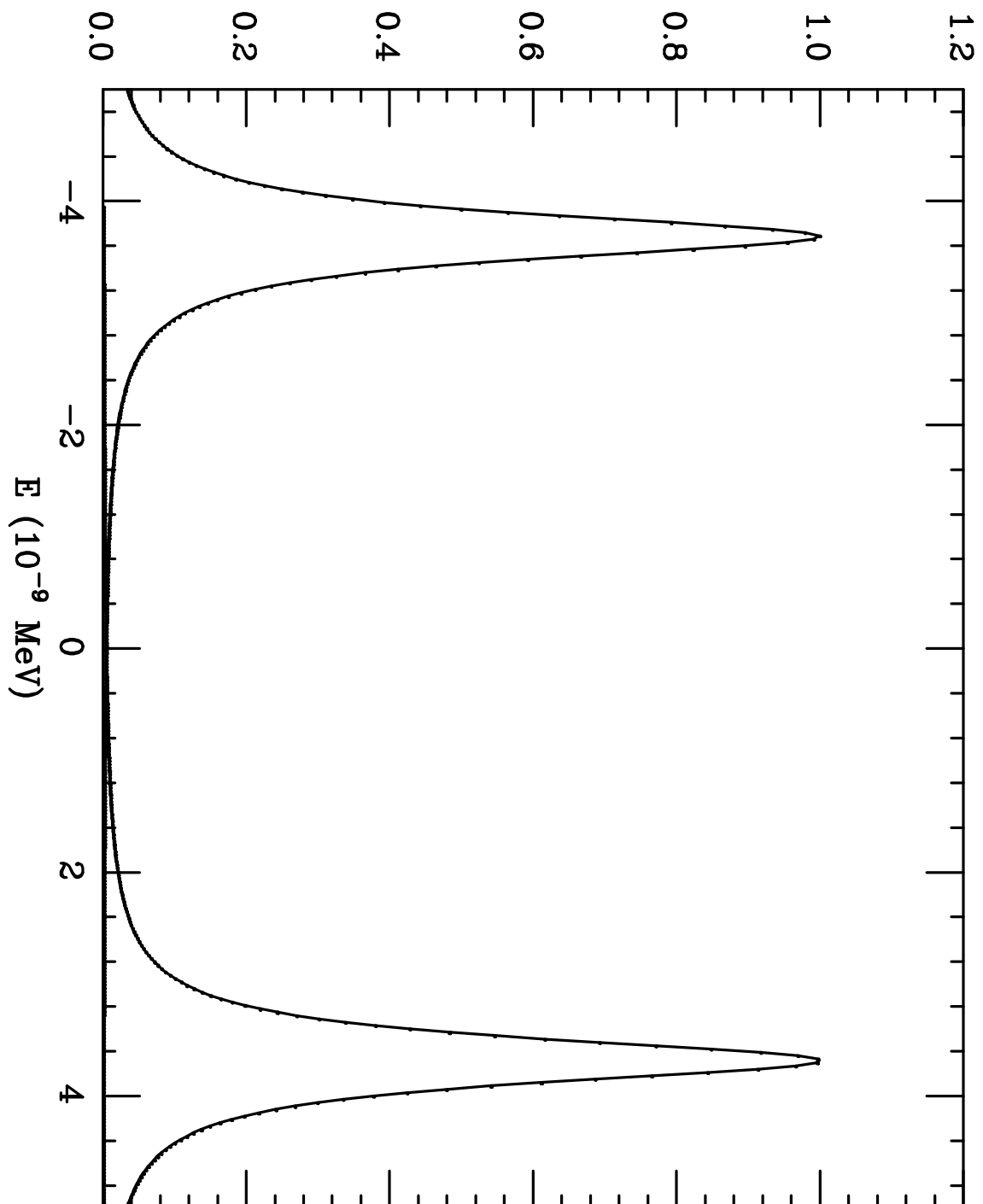
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# $B_d - \bar{B}_d$ Mixing

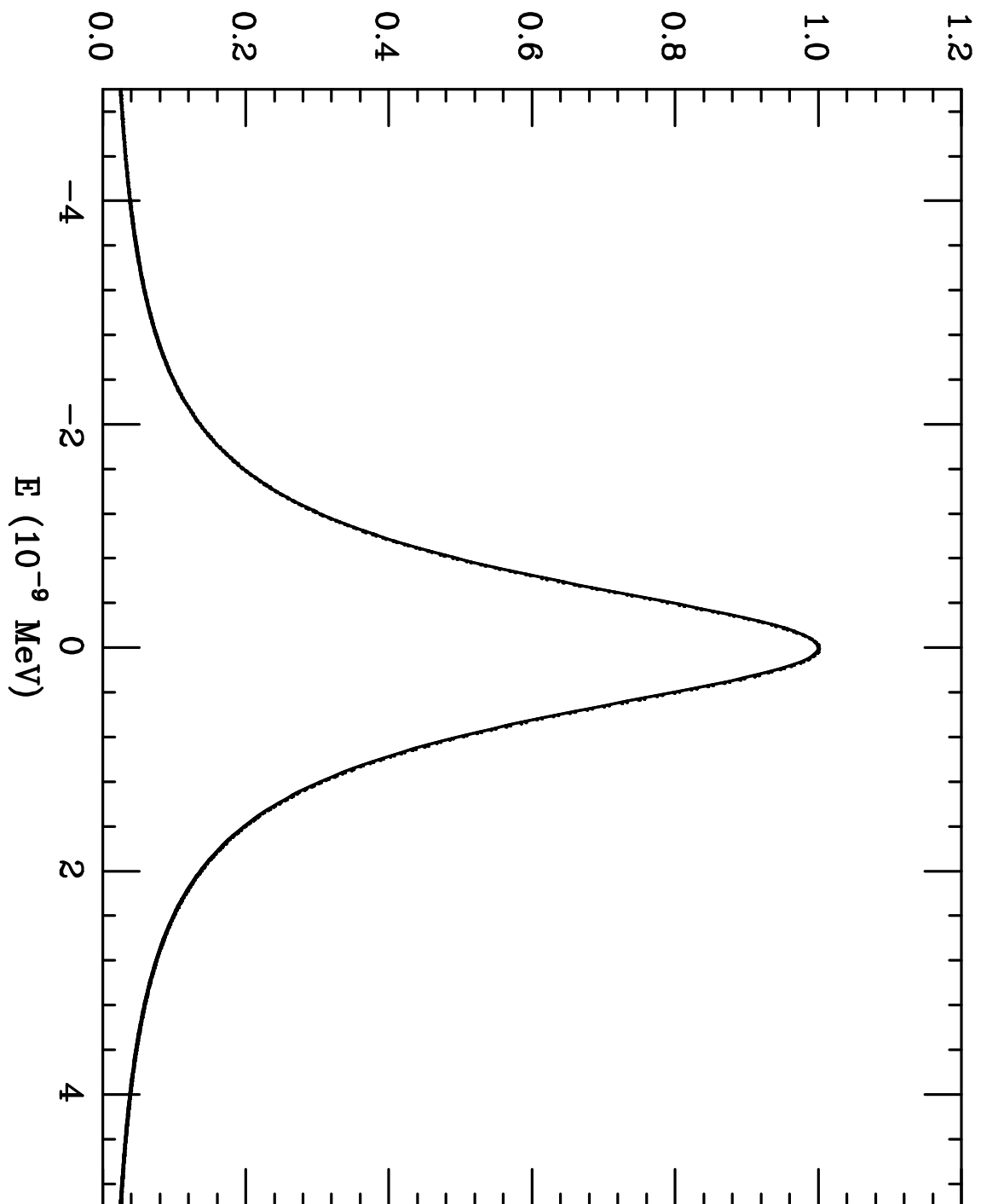




# $B_s-B_s$ Mixing



# D-Dbar Mixing



# K-Kbar Mixing

